The final moment of momentum of the water about the axis of the wheel

per lb. =
$$\frac{(u_3 - {}_3v_r \cos \gamma) r_3}{g}.$$

If ω is the angular velocity of rotation, the work done, U, per lb. per second is given by

$$U = \frac{\omega}{g} \left\{ v_1 \cos \alpha \cdot r_2 - \left(u_3 - {}_3 v_r \cos \gamma \right) r_3 \right\} \text{ft. lbs.}$$

The hydraulic efficiency of the machine, neglecting mechanical losses and disc windage, is then equal to $U \div H$, while the hydraulic efficiency of the runner equals $2 g U \div v_1^2$.

The loss due to skin friction and eddy formation in the buckets

$$= \frac{(2v_r + (u_3 - u_2)\cos\gamma)^2}{2g} \left\{ 1 - k^2 \right\} \text{ft. lbs. per lb.}$$

The loss due to the kinetic energy of discharge

$$=\frac{{v_3}^2}{2g}$$
ft. lbs. per lb.

In an axial flow turbine, $u_2 = u_3$; $r_2 = r_3$, and these expressions become

$$\begin{split} U &= \frac{\omega}{g} \left\{ \left. v_1 \, r_2 \cos \alpha - \left(u_2 - {}_3 v_r \cos \gamma \right) \, r_2 \right\} \right. \\ &= \frac{u_2}{g} \left\{ \left. v_1 \cos \alpha - u_2 + {}_3 v_r \cos \gamma \right\} \, \text{ft. lbs. per lb.} \end{split}$$
 Bucket Loss
$$= \frac{{}_2 v_r^2 \, (1 - k^2)}{2 \, g} \\ &= \frac{{}_2 v_r^2 - {}_3 v_r^2}{2 \, g} \, \text{ft. lbs. per lb.} \end{split}$$

For maximum hydraulic efficiency, assuming the vane angles designed to give entry without shock, the sum of the losses due to bucket friction and to rejection of kinetic energy must be a minimum. In any wheel of normal design, the exit angle γ should be as small as possible. In practice, this angle cannot be made much less than 15° without making the breadth of the wheel at exit abnormally large. Adopting this value, the angles α and β , and the speed u_2 , are capable of relative adjustment. As indicated by the triangle of velocities, if the angle α and the velocity of the jet are fixed, the peripheral speed of the wheel for entry without shock is increased as β is increased. At the same time the relative velocities, and, therefore, the bucket losses, are