which may be raised or lowered as the stream rises, or the axle carried upon pontoons so that the wheel rises automatically with the stream.

181. Poncelet wheel.—The efficiency of the straight blade impulse wheels is very small, due to the large amount of energy lost by shock, and to the velocity with which the water leaves the wheel in the direction of motion.

The efficiency of the wheel is doubled if the blades are of such a form that the direction of the blade at entrance is parallel to the relative velocity of the water and the blade, as first suggested by Poncelet, and the water is made to leave the wheel with no

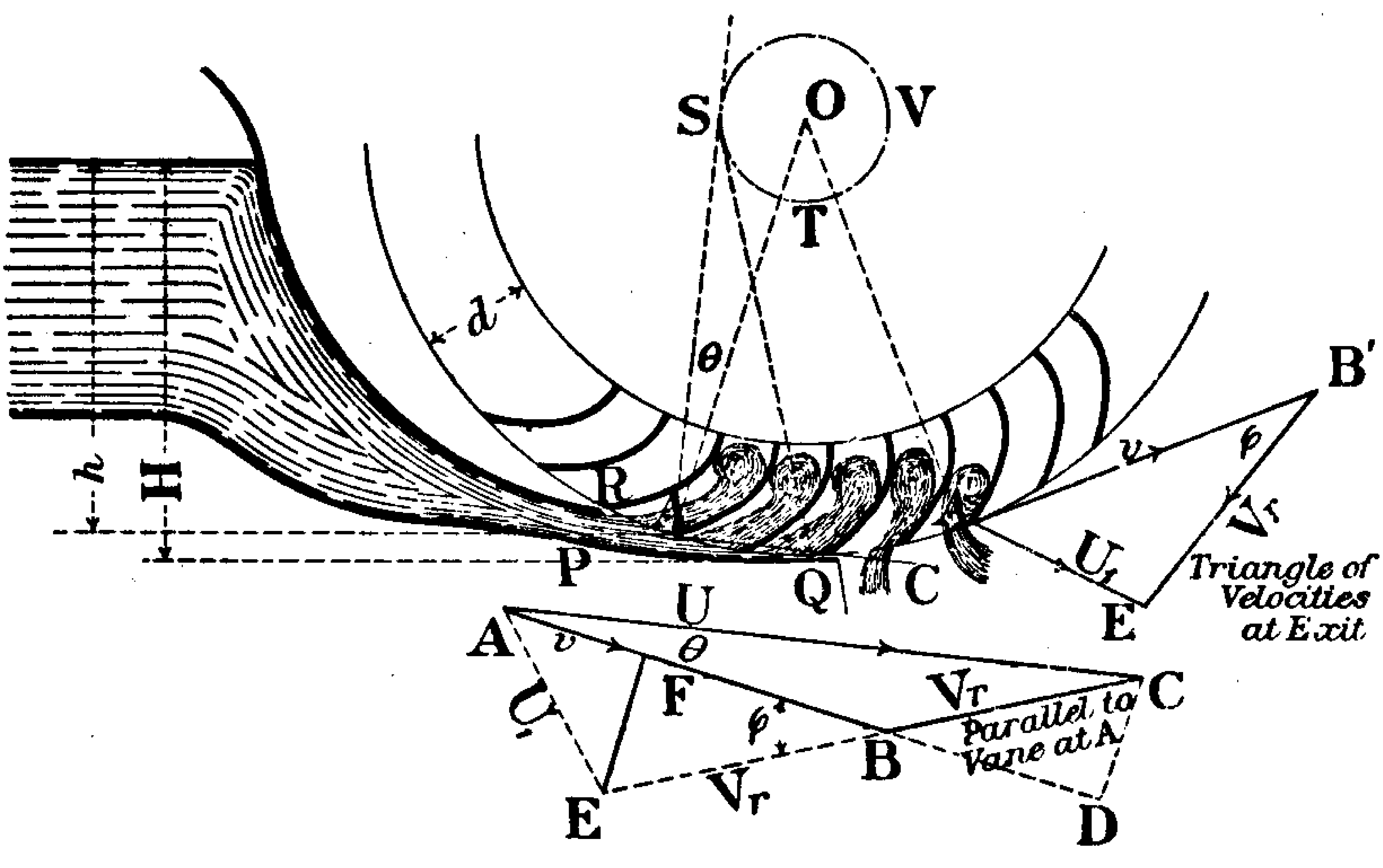

Fig. 292. Undershot Wheel.

component in the direction of motion of the periphery of the wheel.

Fig. 292 shows a Poncelet wheel. Suppose the water to approach the edge A of a blade with a velocity U making an angle θ with the tangent to the wheel at A.

Then if the direction of motion of the water is in the direction AC, the triangle of velocities for entrance is ABC.

The relative velocity of the water and the wheel is V_r, and if the blade is made sufficiently deep that the water does not overflow the upper edge and there is no loss by shock and by friction, a particle of water will rise up the blade a vertical height

$$h_1 = \frac{V_r^2}{2g}.$$

It then begins to fall and arrives at the tip of the blade with the velocity V_r relative to the blade in the inverse direction BE.

The triangle of velocities for exit is, therefore, ABE, BE being equal to BC.

The velocity with which the water leaves the wheel is then

$$AE=U_1.$$

It has been assumed that no energy is lost by friction or by shock, and therefore the work done on the wheel is

$$\frac{U^2}{2g}-\frac{U_1^2}{2g},$$

and the theoretical hydraulic efficiency * is

$$E=\frac{\frac{U^2}{2g}-\frac{U_1^2}{2g}}{\frac{U^2}{2g}}$$

$$=1-\frac{U_1^2}{U^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

This will be a maximum when U_1 is a minimum.

Now since BE=BC, the perpendiculars EF and CD, on to AB and QB produced, from the points E and C respectively, are equal. And since AC and the angle θ are constant, CD is constant for all values of v, and therefore FE is constant. But AE, that is U_1, is always greater than FE except when AE is perpendicular to AD. The velocity U_1 will have its minimum value, therefore, when AE is equal to FE or U_1 is perpendicular to v.

* In what follows, the terms theoretical hydraulic efficiency and hydraulic efficiency will be frequently used. The maximum work per lb. that can be utilised by any hydraulic machine supplied with water under a head H, and from which the water exhausts with a velocity u is $H-\frac{u^2}{2g}$. The ratio

$$\frac{H-\frac{u^2}{2g}}{H}$$

is the theoretical hydraulic efficiency. If there are other hydraulic losses in the machine equivalent to a head h_f per lb. of flow, the hydraulic efficiency is

$$\frac{H-\frac{u^2}{2g}-h_f}{H}.$$

The actual efficiency of the machine is the ratio of the external work done per lb. of water by the machine to H.

The triangles of velocities are then as in Fig. 293, the point B bisects AD, and

$$v=\tfrac{1}{2}\mathrm{U}\cos\theta.$$

For maximum efficiency, therefore,

$$v=\tfrac{1}{2}\mathrm{U}\cos\theta.$$

The efficiency can also be found by considering the change of momentum.

The total change of velocity impressed on the water is CE, and the change in the direction of motion is therefore FD, Fig. 292.

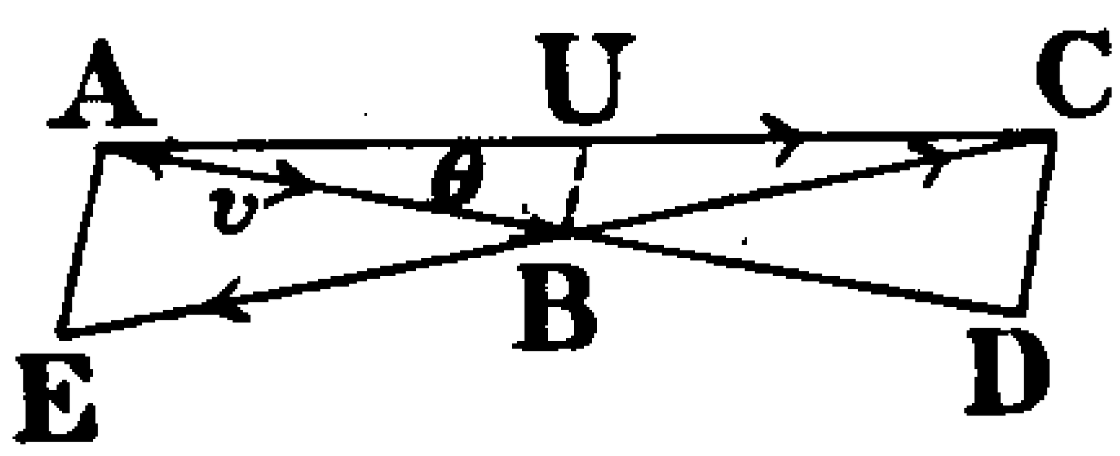

Fig. 293.

And since BE is equal to BC, FB is equal to BD, and therefore,

$$\mathrm{FD}=2(\mathrm{U}\cos\theta-v).$$

The work done per lb. is, then,

$$\frac{2(\mathrm{U}\cos\theta-v)}{g}.v,$$

and the efficiency is

$$\mathrm{E}=\frac{2(\mathrm{U}v\cos\theta-v^2)}{g\left(\frac{\mathrm{U}^2}{2g}\right)}$$

$$=\frac{4(\mathrm{U}v\cos\theta-v^2)}{\mathrm{U}^2} \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Differentiating with respect to v and equating to zero,

$$\mathrm{U}\cos\theta-2v=0,$$

or

$$v=\tfrac{1}{2}\mathrm{U}\cos\theta.$$

The velocity U_1 with which the water leaves the wheel, is then perpendicular to v and is

$$\mathrm{U}_1=\mathrm{U}\sin\theta.$$

Substituting for v its value $\tfrac{1}{2}\mathrm{U}\cos\theta$ in (2), the maximum efficiency is $\cos^2\theta$.

The same result is obtained from equation (1), by substituting for U_1, $\mathrm{U}\sin\theta$.

The maximum efficiency is then

$$\mathrm{E}=1-\frac{\mathrm{U}^2\sin^2\theta}{\mathrm{U}^2}=\cos^2\theta.$$

A common value for θ is 15 degrees, and the theoretical hydraulic efficiency is then 0·933.

This increases as θ diminishes, and would become unity if θ could be made zero. If, however, θ is zero, U and v are parallel and the tip of the blade will be perpendicular to the radius of the wheel.

This is clearly the limiting case, which practically is not realisable, without modifying the construction of the wheel. The necessary modification is shown in the Pelton wheel described on page 520.

The actual efficiency of Poncelet wheels is from 55 to 65 per cent.

Form of the bed. Water enters the wheel at all points between Q and R, and for no shock the bed of the channel PQ should be made of such a form that the direction of the stream, where it enters the wheel at any point A between R and Q, should make a constant angle θ with the radius of the wheel at A.

With O as centre, draw a circle touching the line AS which makes the given angle θ with the radius AO. Take several other points on the circumference of the wheel between R and Q, and draw tangents to the circle STV. If then a curve PQ is drawn normal to these several tangents, and the stream lines are parallel to PQ, the water entering any part of the wheel between R and Q, will make a constant angle θ with the radius, and if it enters without shock at A, it will do so at all points. The actual velocity of the water U, as it moves along the race PQ, will be less than $\sqrt{2gH}$, due to friction, etc. The coefficient of velocity k_v in most cases will probably be between 0·90 and 0·95, so that taking a mean value for k_v of 0·925,

$$U=0{\cdot}925\sqrt{2gH}.$$

The best value for the velocity v taking friction into account. In determining the best velocity for the periphery of the wheel no allowance has been made for the loss of energy due to friction in the wheel.

If V_r is the relative velocity of the water and wheel at entrance, it is to be expected that the velocity relative to the wheel at exit will be less than V_r, due to friction and interference of the rising and falling particles of water.

The case is somewhat analogous to that of a stone thrown vertically up in the atmosphere with a velocity v. If there were no resistance to its motion, it would rise to a certain height,

$$h_1=\frac{v^2}{2g},$$

and then descend, and when it again reached the earth it would have a velocity equal to its initial velocity v. Due to resistances, the height to which it rises will be less than h_1, and the velocity with which it reaches the ground will be even less than that due to falling freely through this diminished height.

Let the velocity relative to the wheel at exit be nV_r, n being a fraction less than unity.

The triangle of velocities at exit will then be ABE, Fig. 294. The change of velocity in the direction of motion is GH, which equals

$$\begin{aligned}BH+GB&=BH(1+n)\\&=(1+n)\,(U\cos\theta-v).\end{aligned}$$

If the velocity at exit relative to the wheel is only nV_r, there must have been lost by friction, etc., a head equal to

$$\frac{V_r^2}{2g}(1-n^2).$$

The work done on the wheel per lb. of water is, therefore,

$$\frac{\{(1+n)(U\cos\theta-v)\}v}{g}-\frac{V_r^2}{2g}(1-n^2).$$

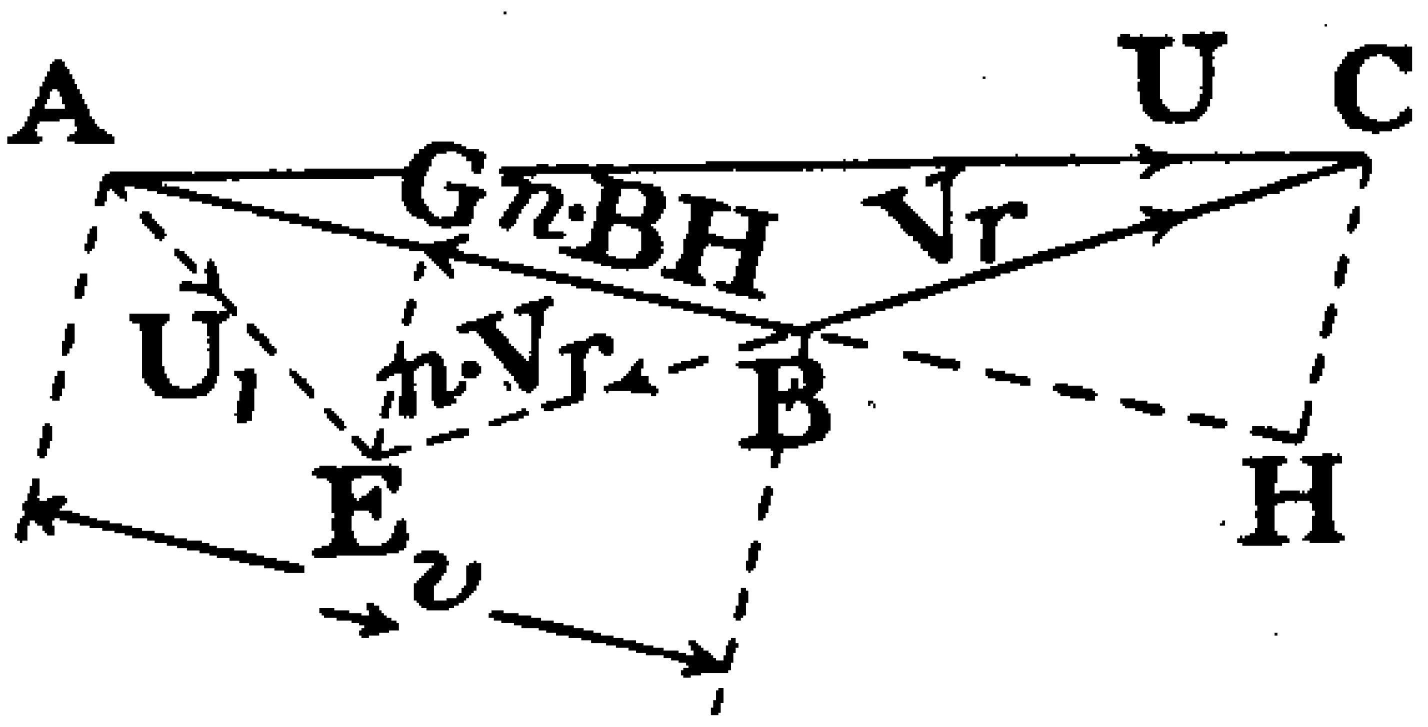

Fig. 294.

Let $(1-n^2)$ be denoted by f, then since

$$V_r^2=BH^2+CH^2=(U\cos\theta-v)^2+U^2\sin^2\theta,$$

the efficiency

$$e=\frac{\{(1+n)(U\cos\theta-v)\}\dfrac{v}{g}-\dfrac{f}{2g}\{(U\cos\theta-v)^2+U^2\sin^2\theta\}}{\dfrac{U^2}{2g}}.$$

Differentiating with respect to v and equating to zero,

$$2(1+n)U\cos\theta-4(1+n)v+2Uf\cos\theta-2vf=0,$$

from which

$$\begin{aligned}v&=\frac{\{1+n)+f\}U\cos\theta}{f+2(1+n)}\\&=\frac{(2+n-n^2)\,U\cos\theta}{3-n^2+2n}.\end{aligned}$$

If f is now supposed to be 0·5, *i.e.* the head lost by friction, etc., is $\frac{0{\cdot}5V_r^2}{2g}$, n is 0·71 and

$$v=\cdot56U\cos\theta.$$

If f is taken as 0·75,

$$v=0{\cdot}6U\cos\theta.$$

Dimensions of Poncelet wheels. The diameter of the wheel should not be less than 10 feet when the bed is curved, and not less than 15 feet for a straight bed, otherwise there will be considerable loss by shock at entrance, due to the variation of the angle θ which the stream lines make with the blades between R and Q, Fig. 292. The water will rise on the buckets to a height nearly equal to $\frac{V_r^2}{2g}$, and since the water first enters at a point R, the blade depth d must, therefore, be greater than this, or the water will overflow at the upper edge. The clearance between the bed and the bottom of the wheel should not be less than $\frac{3}{8}''$. The peripheral distance between the consecutive blades is taken from 8 inches to 18 inches.

Horse-power of Poncelet wheels. If H is the height of the surface of water in the penstock above the bottom of the wheel, the velocity U will be about

$$0{\cdot}92\sqrt{2gH},$$

and v may be taken as

$$0{\cdot}55\times0{\cdot}92\sqrt{2gH}=0{\cdot}5\sqrt{2gH}.$$

Let D be the diameter of the wheel, and b the breadth, and let t be the depth of the orifice RP. Then the number of revolutions per minute is

$$n=\frac{0{\cdot}5\sqrt{2gH}}{\pi . D}.$$

The coefficient of contraction c for the orifice may be from 0·6, if it is sharp-edged, to 1 if it is carefully rounded, and may be taken as 0·8 if the orifice is formed by a flat-edged sluice.

The quantity of water striking the wheel per second is, then,

$$Q=0{\cdot}92ctb\sqrt{2gH}.$$

If the efficiency is taken as 60 per cent., the work done per second is $0{\cdot}6\times62{\cdot}4QH$ ft. lbs.

The horse-power N is then

$$N=\frac{34{\cdot}5 . c . t . b\sqrt{2gH} . H}{550}.$$

182. Turbines.—Although the water wheel has been developed to a considerable degree of perfection, efficiencies of nearly 90 per cent. having been obtained, it is being almost entirely superseded by the turbine.

The old water wheels were required to drive slow-moving machinery, and the great disadvantage attaching to them of having a small angular velocity was not felt. Such slow-moving wheels are, however, entirely unsuited to the driving of modern